Mixed flow modelling produced by pressure fronts from upstream and downstream extremes

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Abstract

Most of numerical procedures used in hydraulic design are able to compute free-surface or pressure flow. But mixed flow is a problem that appears in some hydraulic structures and different procedures have been developped to deal with this problem. The problem that has usually focussed the atention of many researchers is the one surge front, moving upstream or downstream. Different methods have been tested to reproduce the pressure field values, surge celerity, and so on. But another problem that can appear in a sewer network is the transition from free-surface to pressure flow at both ends of a pipe. With the purpose to analyse this phenomenon, an experimental facility was set-up to carry different tests in order to gain knowledge about the problem (qualitative objectives) and reproduce by means of numerical modelling the observed results. Different numerical approaches were tested and finally, an approach based on the full unsteady flow equations, both for pressure and free-surface flow, considering the MOC (Method of Characteristics) provided the best results.

Introduction

Transient between free-surface and pressure flow is an important feature in sewer network design. Most criterias adopt the free surface pattern for an specified discharge corresponding to a return period. But obviously some day a discharge greater than the used in the design calculation could arrive, producing a pressure flow wave that travels upstream or downstream.

Most of numerical procedures are able to compute free-surface or pressure flow. Different codes have been developped to deal with this problem. But in most cases, the problem that has been analysed is the travelling of a surge front, upstream or downstream, produced by the transition from free-surface to pressure flow in one of the extremes of the pipe. The submergence of this extreme can be produced by a sudden increase of discharge or a blockage downstream.

The problem that has usually focussed the atention of many researchers is the one surge front, moving upstream or downstream. Different methods have been tested to reproduce the pressure field values, surge celerity, and so on. One of the common conclussions of the research was that this pressure wave is much more similar with a gravity wave than to a pressure wave. Wave celerities observed were of the order of meters per second or tens of meters per second, that is, one or two magnitude orders less than a pressure wave celerity.

But another problem that can appear in a sewer network is the transition from free-surface to pressure flow at both ends of a pipe. An increase in the discharge at the extremes, due to the irregular spatial rainfall distribution or other situations, can lift the water level to the pipe sofit, creating two different pressure fronts that move to the middle of the pipe, and shock there. With the purpose to analyse this phenomenon, an experimental facility was set-up to carry different tests in order to gain knowledge about the problem (qualitative objectives) and reproduce by means of numerical modelling the observed results. One of the most significant remarks in our consideration is the fact that it was not possible to reproduce the process as the superposition of two fronts, and the significant pressure values observed in some tests. Specially when the air trapped is not easily released, pressure values of 20 times the diameter were found. In this paper we make a description of the set-up used and the experimental and numerical results obtained.

Experimental work

A new set-up was built to analyse the process. As it is shown in figure 1, it is formed by a circular pipe of 153 mm, 12 meters long. Upstream section is conected to a head tank that feeds the model. An electromagnetic flow measurement is included to measure the discharge entering the pipe. A partially closed butterfly valve is located upstream the pipe, in such a way that a sudden opening of the pipe can produce an increase in discharge at the upstream end of the pipe. At the downstream end another partially closed butterfly valve is placed, in order to obtain different initial conditions (backwater curves) and to create a rapid closure at downstream end. As a security device, a surge tank

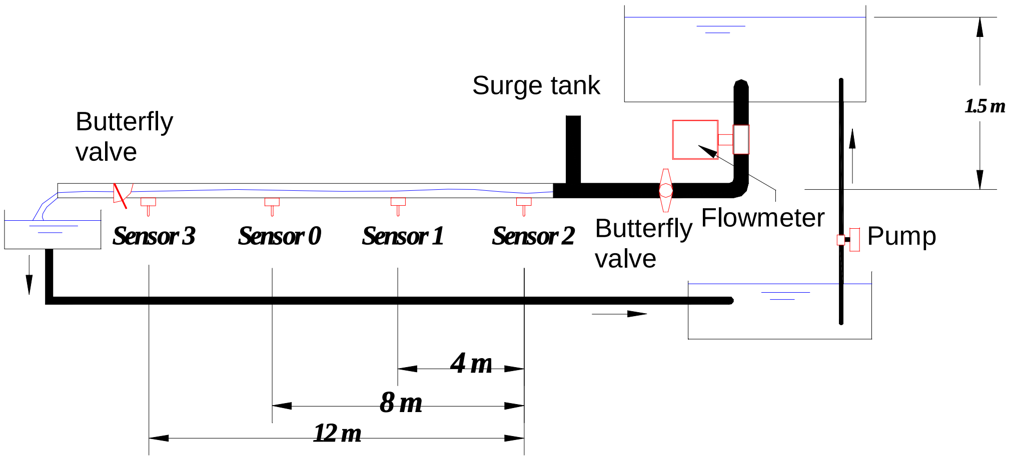

Figure 1.- Experimental set-up

was placed upstream, to avoid too much high pressures in case of malfunctioning of the installation. Four pressure transducers were installed separated two meters. Pressure signals were recorded with a 40 Hz frequency through a data acquisition card, and a Lab-view based data acquisition program.

Pipe slope as 0.001, and a Manning roughness coefficient of 0.010 was measured. Experiments were conducted with initial discharges from 2 to 8 l/s. Subcritical flow was the initial regime condition in all the tests. Downstream boundary conditions ranged from 8 to 12 cm water depths. Lower water depth values did not produce the transition to pressure flow, but an increase of water depth. Greater water depth values did not show a pressure front but almost an instantaneously transition to the pressure flow of the whole pipe.

Experimental results

Three different types of experiments were conducted:

a) Sudden closure of downstream end valve, creating a pressure front moving upstream
b) Sudden opening of upstream valve, producing an increase of discharge, and a pressure wave moving downstream
c) Simultaneous operation of upstream and downstream valves, creating two pressure waves moving downstream and upstream, meeting at the center of the pipe

For the first type of experiments, in some cases a pneumatic perturbation was observed. The water surface showed no perturbartion but an increase of the pressure was detected, that dissipated inmediately. This was observed specially in the most rapid closing valve experiments, with high water levels. The wave celerity observed was close to the sound celerity wave.

The characteristics of the pressure front agrees with all the experimental results of previous authors. Pressure values were of the order of the pipe diameter. Wave celerity measurements showed a good relationship with the downstream water depth. These results can be reproduced quite well with all the different numerical approaches (rigid water column, Preissmann slot or MOC).

Experiments of the type b) were very similar to the type a). The pressure wave characteristics are similar of those observed in the previous ones. The results were well reproduced too. All the mathematical approaches were enough good.

Experiments of type c) were totally different to those observed in cases a and b. From the test data values analysed, it was concluded that it can not be explained as a superposition of two fronts like a-type or b-type. These cases were the main subject of this study

Results Analysis

There is a lot of literature about the transition from free surface to pressurized flow by only one extreme, hardly ever downstream, creating an interface between the pressurized and the free surface region that will advance upstream. In these cases, it can be observed how this interface advance increases the pressure head in a moderated form, until the height of the water column is approximately equal to the double of the height of the pipe, in the case that the transition from pressurized flow to free surface is produced from downstream to upstream. And in the case that the transition from pressurized flow to free surface is produced from upstream to downstream, the height of the column of water is approximately equal to the height of the pipe multiplied by four. Notice that these pressure values depends on the experimental conditions, and this is the behaviour observed in our case.

In the case that there are two transitions from pressurized to free surface flow, there are two surges advancing one from upstream and the other one from downstream and the pressure is much more important than when there is only one transition, so this case will be more dangerous.

From our experimental results we can distinguish two cases:

(III.1) First of all, the closing of the downstream valve is produced, and the opening of the upstreamvalve is produced later. In this case the highest-pressure value is approximately equal to the height of the pipe multiplied by ten.

(III.2) The opening of the upstream valve is the first produced, and the closing of the downstream valce is produced later. This case is the most dangerous; because the highest pressure value is approximately equal to the height of the pipe multiplied by twenty-five.

From the experimental results we can reject the principle of superposition, so we will have a shock of surges.

Mathematical description of free surface and pressure flow

The free surface unsteady gradually varied flow equations were proposed in 1871 by A.J.C. Barre de Saint Venant. By establishing the conservation of mass and momentum quantities in the flow in an open channel, two quasilinear partial differential equations can be obtained. These equations are:

$$\frac{\partial \mathrm{y}}{\partial \mathrm{t}} + \mathrm{v}\frac{\partial \mathrm{y}}{\partial \mathrm{x}} + \frac{\mathrm{c}^2}{\mathrm{g}}\frac{\partial \mathrm{v}}{\partial \mathrm{x}} = 0 \qquad (1)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x} = g\left(S_0 - S_f\right) \qquad (2)$$

Where $c = \sqrt{g\frac{A}{T}}$ (3) is the celerity of the gravity wave (the propagation rate of a small disturbance). A is the cross-sectional area of the flow, T is the topwith of the flow, S_f is the gradient energy slope and S_o is the bottom pipe slope. c is always smaller than10 m/s in our case. The unknown variables in free surface are velocity and depth (v,y).The equations for unsteady pressurized flow are:

$$\frac{\partial y}{\partial t} + v\frac{\partial y}{\partial x} + \frac{a^2}{g}\frac{\partial v}{\partial x} = 0 \qquad (3)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x} + g(S_f - S_0) = 0 \qquad (4)$$

where a is the celerity of the pressure wave, which it is assumed to be constant, the propagation velocity of a wave inside the pressurized pipe with some air trapped. This velocity of the pressure wave is much bigger than the velocity of gravity wave. It is about 1000 m/s when the pipe is completely filled by water, and this value decreases when the air trapped increases to be practically 0 m/s when there is no water in the pipe. The unknown variables in the pressurized flow equations are the velocity and the pressure (v,y).

Numerical model proposed

We are going to use the Method of Characteristics (MOC) formulation for the numerical integration of differential flow equations in the different regions, free surface and the pressurized flow. If the Courant condition is strictly guaranteed, numerical stability is assured. We are going to apply the equations of mass conservation, and the momentum conservation in the intersections of pressurized and free surface regions (transitions).

Firstly, the initial steady state water-surface profile is computed as in a natural channel by integrating the steady GVF differential equation using a fourth order Runge-Kutta method. The boundary conditions to integrate the GVF equation are the initial flow rate (Q) and the initial water depth or piezometric head measured at downstream end.

MOC solves the two differential equations in partial derivatives by means of the transformation in two equations in total derivatives that are valid along one different characteristic line. These equations are valid along their associated characteristic line. The first one is called positive characteristic line (C^+), and the second one negative characteristic line (C^-). By discretizing the transformed equations we can easily obtain the velocity and the pressure at point P as:

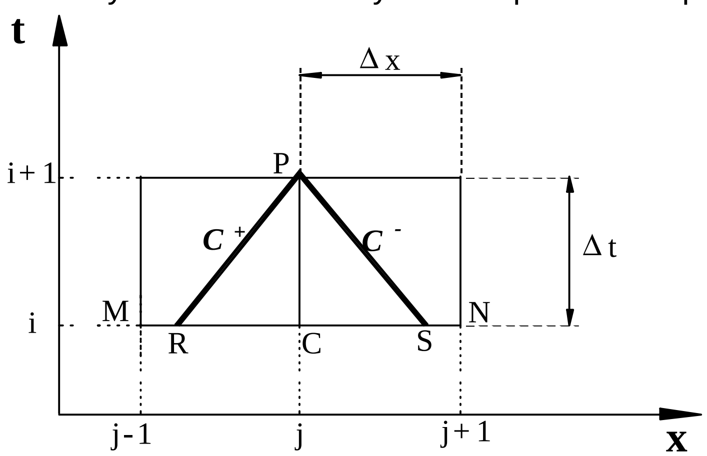

Figure 2.- Fixed grid system for subcritical open channel flow

$$y_P = \frac{1}{c_R + c_S}\left[y_R c_S + y_S c_R + c_S c_R \left(\frac{v_R - v_S}{g} - \Delta t \cdot \left(S_{fR} - S_{fS} \right) \right) \right] \quad (5)$$

$$v_P = v_R - \frac{g}{c_R}\left(y_P - y_R \right) + g \cdot \Delta t \cdot \left(S_0 - S_{fR} \right) \quad (6)$$

where the values of variables at the points R and S can be obtained by linear interpolation. If we are at a boundary them one of the characteristic equations is outside of our boundary of problem, so we are going to need a boundary condition that will be the open channel depth in this boundary. When this open channel depth at the boundary is bigger than the height of the pipe the surge is created. The surge is a transition from pressurized flow to free surface flow.

To solve the pressurized flow, we make a similar process that we did with the free surface flow. The Courant condition for the pressurized flow is $\Delta t < \Delta x/(v+a)$ The parameter a is much greater than c so this is the stability condition that we are going to apply for the whole grid. Δt is done from the experiments so we can easily obtain the Δx for our grid.

In the transition regions of two different flows, they are no valid the characteristic equations that across the interface trajectory. In this regions the equations that we have applied are the conservation of mass and momentum quantities and the characteristic equation for the pressurized flow that not cross the interface trajectory.

For example, in the case of surge that advances from upstream to downstream: The free surface side of the interface can be solved separately to find the velocity and the pressure at P1, by using the linear equations C1- and C1+.

In order to solve the pressurized side of the interface we are going to apply:

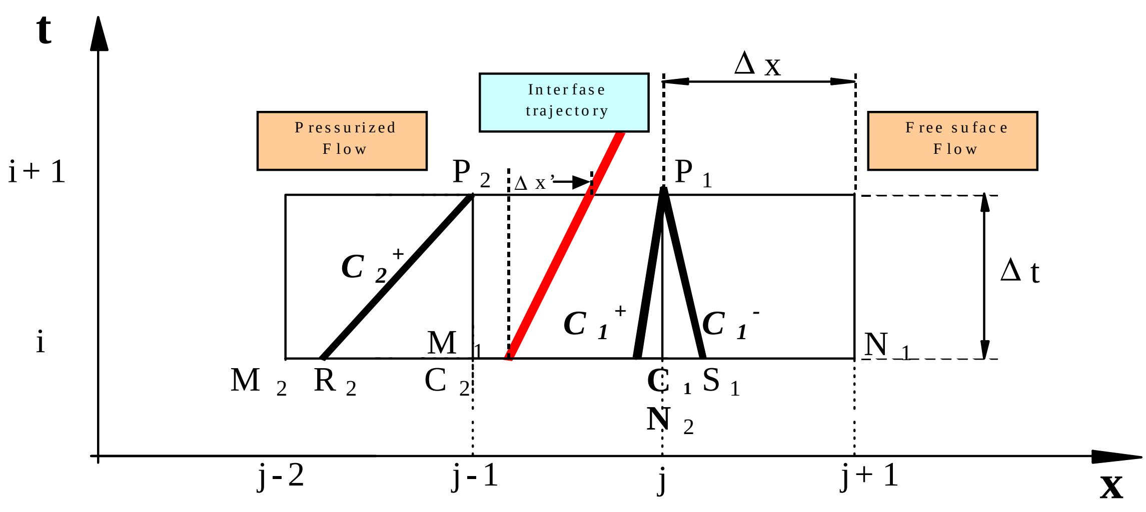

Figure 3.- Grid system near an interface advancing downstream.

1. Continuity equation.

$$A_{P2}\left(v_{P2} - w\right) = A_{P1}\left(v_{P1} - w\right) \quad (7)$$

2. Conservation momentum quantities.

$$\left(y_{P2}-\bar{y}_{cmP2}\right)A_{P2}-\bar{y}_{cmP1}A_{P1}=\left(v_{P1}-w\right)\frac{A_{P1}}{g}\left(v_{P1}-v_{P2}\right) \qquad (8)$$

where y_{cm} is the distance from water to center of gravity of cross-sectional area.

3. Pressurized positive characteristic equation.

$$v_{P2}-v_{R2}+\frac{g}{a}\left(y_{P2}-y_{R2}\right)=\Delta t\cdot g\cdot\left(S_0-S_{fR2}\right) \qquad (9)$$

The solution of these equations gives a quadratic equation for the interface velocity:

$$w^2+B\cdot w+C=0 \qquad (10)$$

where $$B=-2\cdot v_{P1}+\frac{A_{P2}}{A_{P1}}\cdot a \qquad (11)$$

$$C=v_{P1}{}^2-\frac{A_{P2}}{A_{P1}-A_{P2}}\left(\bar{y}_{cmP1}g+v_{P1}a\right)+\frac{A_{P2}{}^2}{A_{P1}\left(A_{P1}-A_{P2}\right)}\left[v_{R2}a+(y_{R2}-\bar{y}_{cmP2})g-a\Delta t\cdot g(S_{fR2}-S_0)\right]$$

Only the positive root make physical sense, so that from the quadratic formula:

$$w=\frac{-B+\sqrt{B^2-4C}}{2} \qquad (12)$$

The model calculates the velocity at point P2 with the continuity equation as:

$$v_{P2}=w+\frac{A_{P1}}{A_{P2}}\cdot\left(v_{P1}-w\right) \qquad (13)$$

and the pressure on the same point P2 from the momentum equation as:

$$y_{P2}=y_{R2}+a\cdot\left(\frac{v_{P2}-v_{R2}}{g}-\Delta t\cdot\left(S_0-S_{fR2}\right)\right) \qquad (14)$$

The new interface position is found from the kinematic condition according to:

$$\Delta x'=w\cdot\Delta t \qquad (15)$$

Notice that the fluid in the moving co-ordinate system is moving from right to left across the shock from the pressurized to the free surface flow. The following condition therefore holds: a+v2 > w > c + v1 (25). This inequality guarantees the applicability of the C2+, C1+ and C1- characteristic equations.

When we are over the boundary, the C2+ characteristic equation becomes not appliable, so we need to substitute it with one boundary condition, that is, the pressure at the upstream end of the system. In the other case, when the surge advances from downstream to upstream, we have applied the same method exposed. Notice that in this case the surge advance is opposed to the water advance.

Surge crossing

The surge crossing appears when the distance between surges is less than one Δx. In this case the characteristic equations for the free surface flow becomes not applicable because there are no points of free surface flow in our system grid. But we know that between the surges there is still a free surface flow. In order to solve the flow, we need to introduce a new point X (which is in free surface flow condition), and now we can apply mass and momentum conservation equations and the corresponding characteristic equation for the pressurized flow. The values of the velocity and the depth at this point X are assumed to be constant and equals to those of the last free surface point computed. We make this process until the interfaces trajectory has an intersection. After that moment all the points are assumed to be in pressurized flow and computed as so.

Influence of the air trapped in the pressure wave speed

Wylie and Streeter give experimental results which demonstrate that the pressure wave speed drops from about 1000 m/s for zero air content to about 200 m/s for an air content of 1.0% and to about 0 m/s when there is practically no water in the pipe. This fact makes necessary to introduce a non uniform value for the pressure wave speed, which was assumed to be constant in the studies about the propagation of only one surge. They give also a formula to calculate the value of "a" but unfortunately this formula is not useful from the experimental data. So we have used the value of the pressure wave speed as a calibration parameter for the model.

In order to get a good behaviour of the model; we need to reduce the value of the pressure wave speed after the shock of surges. This can be explained by the effect of the air trapped during the shock wave, because there is no exit for the air inside the pipe. As we have said there is a non uniform value for the pressure wave speed, but we have succeed to reproduce this champ with only two values of a. One before the shock of surges and the other one after the shock. The first value has been fixed to about 100 m/s. This value assumes that the air trapped in the surge advance will be about 1%.

For the second value, we can distinguish two cases:

(III.1) The closing of the downstream valve is the first produced: In this case the pressure wave speed drops to 10 m/s in our case.
(III.2) The opening of the upstream valve is the first produced: The pressure wave speed drops to 15 m/s in our experiments.

This second value needs to be fine tuned. The differences between the two cases can be attributed to the effect produced by the surge tank, and that some air can be pulled out when the downstream valve is closed first. The values for the pressure wave speed (a) seems to have a high parallelism with the one of the gravity wave speed (c).

Comparison of experimental and simulated results

As we say before, we need to distinguish two cases depending which surge is the first produced.

Case III.1: The surge advancing to upstream end will be the first produced.

In the picture, we are going to show the result of an experiment of this type. In this test the initial discharge measured was 6.08 l/s and the initial depth at downstream end was equal to 11,1 cm.

The initial value for the pressure wave speed was equal to 100 m/s and the second one used before the surge crossing was equal to 9 m/s. In this case the surge velocity computed is equal in the two surges and it is about 6 m/s.

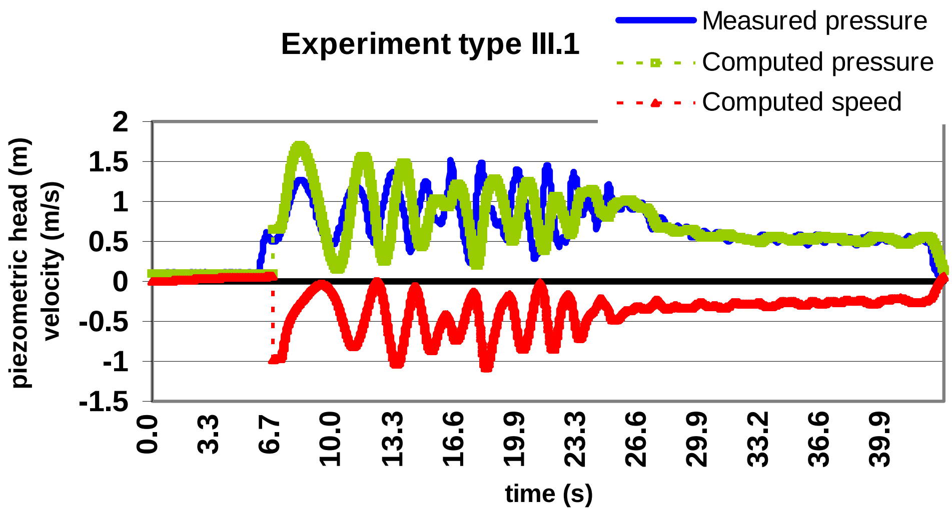

Figure 5.- Simulation results for an experiment type III.1

Case III.2: The surge that advance from upstream to downstream will be the first produced.

In the showed experiment, the initial wealth measured was 3.34 l/s and the initial depth at downstream end was equal to 10,0 cm. The initial value for the pressure wave speed was equal to 90 m/s and the second one used before the shock of surges was equal to 14 m/s.

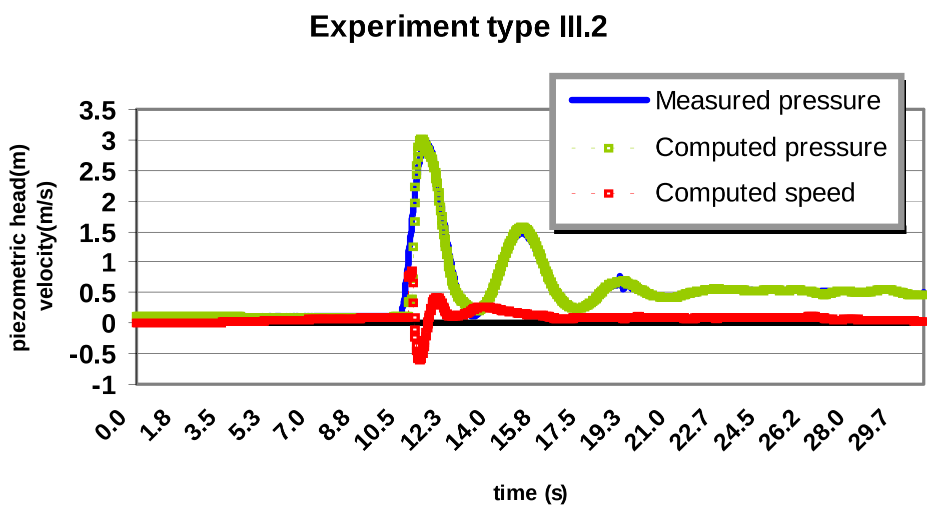

Figure 6.- Simulation results for an experiment type III.2

In this case the surge velocity computed is not equal for the two surges and it is about to 3,5 m/s for the surge that initiates downstream and it is about 9 m/s for the other one. The first oscillations of the water speed value during the shock of surges remember the "water hammer" phenomena.

Summary and conclusions

In this paper a set of experimental tests is presented analysing the combination process of two surge fronts moving in a pipe with a significant trapped air inside. After an experimental campaign, the results showed a different behaviour as that of a single surge front moving upstream or downstream. The combination of both surge fronts produces a much more intense pressure field, different as the observed in single front tests. A numerical model based on the method of characteristics is able to reproduce the measurements. Wave celerity was selected as the parameter to reproduce observed pressures and two different values were used, one before and the second after the front combination.

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